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Performance evaluation and finding target using multi-objective programming in data envelopment analysis with stochastic data

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Abstract. In real world applications, we face stochastic data such as costs, prices, and production quantities that are dependent on various events, such as social factors and politics, inflation, and natural factors, on the other hand, it is very important to evaluate the performance of the decision-making units and to find target for inefficient units in the case that inputs and outputs of units have stochastic values, as well as finding Paratoo's solutions in multi-objective programming. In this paper, we propose a multi-objective model with stochastic variables. Due to the use of interactive methods in solving this model, the targets obtained are more practical and closer to reality for inefficient units. And by interacting between the decision maker and the analyst, a target closer to the expectations of the decision maker can be obtained. We use the model to get the target for inefficient units among the 30 public university units.

Key words. Efficiency, multi-objective programming, stochastic data.

1. Introduction

One of the important responsibilities of management in any organization is decision making. The importance of this is to the extent that some management experts like Herbert Simon consider the management to be a matter of decision making [1]. Most managerial decisions are influenced by a variety of quantitative and qualitative factors that often conflict with each other and they try to choose the best option among several options. Mistakes and inaccuracies in decision making require an error. The greater the power and authority of management, the higher the cost of the wrong decision [2–4]. On the other hand, it is important to evaluate the performance of the decision-making units as well as to find the appropriate model for the decision-maker units, but it is necessary to find a model compatible with the community's realities. So here, using the data envelopment analysis, we get the efficiency of each

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decision maker, and then we get an efficient pattern for inefficient units, with which administrators can effectively improve that unit's performance. Hossein Zadeh Lotfi et al. [5], Wolozano et al. [6] and Haddad et al. [4] have been investigating the ineffectual depiction of data envelopment analysis. In the proposed models, the data are definite values, but in real-world applications, we encounter stochastic data such as costs, prices, and production quantities that are dependent on various events, such as social factors and politics, and inflation and natural factors. Therefore, it was necessary to provide analytical models of data coverage for stochastic data.

2. Methodology

2.1. Multi-objective problems with stochastic data

Assume n DMUs with m inputs and s outputs. The input and output vectors of DMU_j, $j = 1,\ldots,n$ are $X_j = (x_{1j},\ldots,x_{mj})$, $Y_j = (y_{1j},\ldots,y_{sj})$. Hosseinzadeh et al. (2009) have proposed the following MOP problem for finding the projection of inefficient units the variable return to scale (VRS) case:

$$\min\left(\alpha_1,\ldots,\alpha_m,\ -\beta_1,\ldots,-\beta_s\right)$$

$$\text{s.t.}\ \sum_{j=1}^{n}\lambda_j x_{ij} \le \alpha_i,\ \forall i \sum_{j=1}^{n}\lambda_j y_{rj} \ge \beta_r, \forall r,\ \sum_{r=1}^{n}\lambda_j = 1\,. \tag{1}$$

Let $\tilde{X}_j = (\tilde{x}_{1j},\ldots,\tilde{x}_{mj})$, $\tilde{Y}_j = (\tilde{y}_{1j},\ldots,\tilde{y}_{sj})$ be stochastic input and output related to DMU_j, $j = 1,\ldots,n$, both having the normal distribution. We assume that$\overline{x}_{ij}$ and $\overline{y}_{ij}$ are the means of the input and output variables, the stochastic version of the model (1) with inequality constraints is

$$\min\left(\alpha_1,\ldots,\alpha_m,-\beta_1,\ldots,-\beta_s\right)\,,$$

$$\text{s.t.}\ p\left(\sum_{j=1}^{n}\lambda_j\tilde{x}_{ij} \le \alpha_i\right) \ge 1-A,$$

$$p\left(\sum_{j=1}^{n}\lambda_j\tilde{y}_{rj} \ge \beta_r\right) \ge 1-A,\ \sum_{r=1}^{n}\lambda_j = 1\,. \tag{2}$$

Model (2) can be converted to a definite model by performing the following operations, for this purpose we first convert the input constraints using the slack variable ε_i into the equation

$$p\left(sum_{j=1}^{n}\lambda_j\tilde{x}_{ij} \le \alpha_i\right) = 1-A+\varepsilon_i \tag{3}$$

and by the slack variables s_i^- we have

$$p\left(\sum_{j=1}^{n}\lambda_j\tilde{x}_{ij} \leq \alpha_i - s_i^-\right) = 1 - A\,, \tag{4}$$

$$\tilde{h}_i = \sum_{j=1}^{n}\lambda_j\tilde{x}_{ij}\,. \tag{5}$$

Since each linear combination of normal stochastic variables itself has a normal distribution, therefore, $\tilde{h}_i \sim N\left(\overline{h}_i, \sigma_i^I(\lambda)\right)$ such that

$$\overline{h}_i = E\left(\tilde{h}_i\right) = E\left(\sum_{j=1}^{n}\lambda_j\tilde{x}_{ij}\right) = \sum_{j=1}^{n}\lambda_j\overline{x}_{ij}\,,$$

$$\left(\sigma_i^I(\lambda)\right)^2 = \operatorname{var}\left(\tilde{h}_i\right) = \operatorname{var}\left(\sum_{j=1}^{n}\lambda_j\tilde{x}_{ij}\right) =$$

$$= \sum_{j=1}^{n}\sum_{k=1}^{n}\lambda_j\lambda_k\operatorname{cov}(\tilde{x}_{ij},\tilde{x}_{ik})G\,.$$

Given the stochastic variables $\tilde{h}_i$, (4) can be rewritten as

$$p\left(\tilde{h}_i \leq \alpha_i - s_i^-\right) = 1 - A,\; p\left(\frac{\tilde{h}_i - \overline{h}_i}{\sigma_i^I(\lambda)} \leq \frac{\alpha_i - s_i^- - \overline{h}_i}{\sigma_i^I(\lambda)}\right) = 1 - A\,.$$

Let $\tilde{z}_i = \frac{\tilde{h}_i - \overline{h}_i}{\sigma_i^I(\lambda)}$ such that $\tilde{z}_i \sim N(0,1)$. Then

$$p\left(\tilde{z}_i \leq \frac{\alpha_i - s_i^- - \overline{h}_i}{\sigma_i^I(\lambda)}\right) = 1 - A,\; \varphi\left(\frac{-\alpha_i + s_i^- + \overline{h}_i}{\sigma_i^I(\lambda)}\right) = A\,,$$

$$-\alpha_i + s_i^- + \overline{h}_i - \varphi^{-1}(A)\sigma_i^I(\lambda) = 0\,.$$

Therefore, the deterministic form of the input constraints of the model (2) is as follows:

$$\sum_{j=1}^{n}\lambda_j\overline{x}_{ij} + s_i^- - \varphi^{-1}(A)\,\sigma_i^I(\lambda) = \alpha_i\,. \tag{6}$$

In the same way, the deterministic form of the output constraints of model (2) is obtained as

$$\sum_{j=1}^{n}\lambda_j\overline{y}_{ij} + s_i^- - \varphi^{-1}(A)\,\sigma_r^O(\lambda) = \beta_r\,. \tag{7}$$

Given the above conditions, the model (2) is proposed as

$$\min\left(\alpha_1,\ldots,\alpha_m,-\beta_1,\ldots,-\beta_s\right)\,,$$
$$\text{s.t.}\ \sum_{j=1}^{n}\lambda_j\overline{x}_{ij}+s_i^- -\varphi^{-1}\left(A\right)\sigma_i^I\left(\lambda\right)=\alpha_i\,,$$
$$\sum_{j=1}^{n}\lambda_j\overline{y}_{ij}+s_i^- -\varphi^{-1}\left(A\right)\sigma_r^O\left(\lambda\right)=\beta_r\,,$$
$$\left(\sigma_i^I(\lambda)\right)^2=\sum_{j=1}^{n}\sum_{k=1}^{n}\lambda_j\lambda_k\mathrm{cov}(\tilde{x}_{ij},\tilde{x}_{ik})G\,. \tag{8}$$

The model (8) is used for multi-objective problems with stochastic inputs and outputs, and the target function can be weighed to solve.

Theorem 1: If $A=0.5$, then the results of the MOP-stochastic model(3) and MOLP model (1) are the same.

Proof: Since $\varphi^{-1}\left(A\right)=0$, it is obvious.

Theorem 2: For each A, MOP-stochastic model (3) has a feasible solution.

Proof: $z_0=(\lambda_k=1,\ \lambda_j=0\ \forall j\neq k,\ \alpha_i=x_{ik}\ \forall i,\ \beta_r=y_{rk}\forall r)$ is the feasible solution of MOP-stochastic model (3).

3. Research methodology

We apply the proposed models for evaluating and obtaining an effective picture for 30 government university units in Iran, and we want to evaluate these university units in level of education. Suppose that at the educational level of each university, there are three entries in the form of a single score, number of faculty and number of students, and two outputs in terms of income and the number of students ready to defend. All inputs and outputs have stochastic values with normal distribution. The mean and their variance are given in Table 1. Since all inputs and outputs are stochastic variables, we use model (3) to obtain the target for all universities. It can be observed in Table 1 that in the case of $A=0.2$, $(\varphi^{-1}(A)=-0.25)$, if the objective function weights are assumed $w_1=w_2=w_3=w_4=w_5=0.2$, according to the decision maker's wish, then DMU_5 is obtained as the target for all DMUs, which is an efficient unit. If the objective function weights are $w_1=0.2,\ w_2=w_3=w_4=0.1,\ w_5=0.5$, then DMU_{21} is the target for all DMUs, which is an efficient unit.

If the decision maker does not have any idea on the weights, we obtain the weights by the entropy method. In this case $w_1=0.32$, $w_2=0.28$, $w_3=0.11$, $w_4=0.19$, $w_5=0.2$ and DMU_{11} is the target for all DMUs, which is an efficient unit.

Table 1. MOP-stochastic model

Possibility level A					Objective function coefficients		MOLP-stochastic model		
w_1	w_2	w_3	w_4	w_5	α_1	α_2	α_3	β_1	β_2
0.4	0.2	0.2	0.2	0.2	36800	83	1680	54500	30
0.4	0.2	0.1	0.1	0.5	8400	89	1390	27700	19
Possibility level A					Objective function coefficients		MOLP-stochastic model		
w_1	w_2	w_3	w_4	w_5	α_1	α_2	α_3	β_1	β_2
0.4	0.32	0.28	0.11	0.19	0.2	40700	78	1920	39000

4. Conclusion

In real world applications, we face stochastic data such as costs, prices, and production quantities that are dependent on various events, such as social factors and politics, and inflation and natural factors. In this paper, we propose a multi-objective model for all inclusions and outcomes that are stochastic. The advantage of multi-objective models is that they can use interactive methods to solve them and obtain an image tailored to the decision maker's or manager's and analyst's point of view, which is closer to reality and more practical, and this method gains the entire Paratoo points brought up. To solve the proposed model, we use an interactive weighing method and the weights are determined by the decision maker. If the decision maker or manager has no idea for it, we determine the weights by entropy method. Investigating the return to scale for non-radian models with stochastic data can be considered as future research.

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